Journal of Zhejiang University SCIENCE A
ISSN 1673-565X (Print); ISSN 1862-1775 (Online)
www.zju.edu.cn/jzus; www.springerlink.com
E-mail: jzus@zju.edu.cn

Evaluating transmission towers potentials during ground faults

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Received Sept. 12, 2007; revision accepted Oct. 30, 2007

Abstract: During ground faults on transmission lines, a number of towers near the fault are likely to acquire high potentials to ground. These tower voltages, if excessive, may present a hazard to humans and animals. This paper presents analytical methods in order to determine the transmission towers potentials during ground faults, for long and short lines. The author developed a global systematic approach to calculate these voltages, which are dependent of a number of factors. Some of the most important factors are: magnitudes of fault currents, fault location with respect to the line terminals, conductor arrangement on the tower and the location of the faulted phase, the ground resistance of the faulted tower, soil resistivity, number, material and size of ground wires. The effects of these factors on the faulted tower voltages have been also examined for different types of power lines.

Key words: Overhead transmission lines, Ground fault, Tower potential

doi:10.1631/jzus.A072206 **Document code:** A **CLC number:** TM75

INTRODUCTION

When a ground fault occurs on an overhead transmission line in a power network with grounded neutral, the fault current returns to the grounded neutral through the tower's structure, ground return path and ground wires. During ground faults, a number of towers near the fault are likely to acquire high potentials to ground. These tower voltages, if excessive, may present a hazard to humans and animals. Since during a ground fault the maximum voltage will appear at the tower nearest to the fault, attention in this study will be focused on that tower, called faulted tower. The objective of this paper is to determine the voltage rise of this faulted tower. In order to do this, some analytical methods are presented, which allow the determination of the ground fault current distribution between neutral conductors and the earth, via the towers, for transmission line ground networks and the voltage rise of this faulted tower.

A phase-to-ground fault that appears on a phase of a transmission line divides the line into two sections, each extending from the fault towards one end of the line. Depending on the number of towers between the faulted tower and the stations, on the distance between the towers, these two sections of the line may be considered infinite, in which case the ground fault current distribution is independent of the termination of the network; otherwise, they must be regarded as finite, in which case the ground fault current distribution may depend greatly on the termination of the network.

First to be considered is the case when the fault appears in the last tower of the transmission line, considering both infinite and finite transmission lines. Then, it will be considered that if the fault appears at any tower of the transmission line, the two sections of the line are finite and it is assumed that the fault is fed from both directions.

The calculation method introduced is based on the following assumptions: impedances are considered lumped parameters in each span of the transmission line, capacitances of the line are neglected, the contact resistance between the tower and the ground wire, and respectively the tower resistance between the ground wire and the faulty phase conductor, are neglected.

The approach used in this paper is based on some methods presented by (Rudenberg, 1959; Verma and Mukhedkar, 1979; Dawalibi and Niles, 1984). This approach can be applied to long lines, short lines, respectively in case where are only few spans between the feeding station and the faulted tower. Rudenberg (1959) developed a model which was valid only for long lines, without taking into account the mutual coupling between the faulted phase and the ground conductor. Verma and Mukhedkar (1979) developed that model by taking into account the mutual coupling, but they treated only the case of long line, too. In this paper, the author improved their model by considering even the case of a short line.

FAULTS ON OVERHEAD LINES

When a ground fault occurs on an overhead transmission line, the fault divides the line into two sections, each extending from the fault towards one end of the line. These two sections of the line may be considered infinite if some certain conditions are met; otherwise, they must be regarded as finite.

Infinite half-line

An infinite half-line can be represented by the ladder network presented in Fig.1. It is assumed that all the transmission towers have the same ground impedance Z_{st} and the distance between towers is long enough to avoid the influence between their grounding electrodes. The impedance of the ground wire connected between two grounded towers, called the self impedance per span, is noted with Z_{cp_d}. Considering the same distance l_d between two consecutive towers and that Z_{cp_d} is the same for every span, then $Z_{cp_d} = Z_{cp} l_d$, where Z_{cp_d} represents the impedance of the ground wire in Ω/km. Z_{cp_m} represents the mutual impedance between the ground wire and the faulted phase conductor, per span (Vintan, 2005).

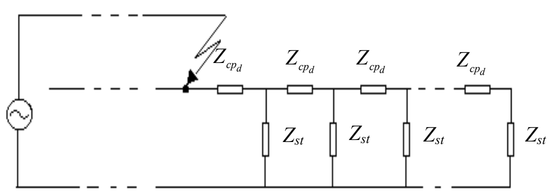

Fig.1 Equivalent ladder network for an infinite half-line

In order to determine the equivalent impedance of the circuit presented in Fig.1, the continuous fractions theory (Edelmann, 1966), already presented in (Vintan, 2005), is applied.

The equivalent impedance seen from the fault location (Fig.1) can be written in the following expression:

$$Z_\infty = Z_{cp_d} + \cfrac{1}{\cfrac{1}{Z_{st}} + \cfrac{1}{Z_{cp_d} + \cfrac{1}{\cfrac{1}{Z_{st}} + \cfrac{1}{Z_{cp_d} + \ldots + Z_{cp_d} + \left(\cfrac{1}{Z_{st}} + \cfrac{1}{Z_{cp_d} + Z_{st}} \right)^{-1}}}}}. \quad (1)$$

Eq.(1) could be written in a recurrent manner using the following equation:

$$Z_\infty = Z_{cp_d} + \left(\frac{1}{Z_{st}} + \frac{1}{Z_{1\infty}} \right)^{-1}. \quad (2)$$

From this expression, the next two-degree equation can be obtained:

$$Z_\infty^2 - Z_{cp_d} Z_\infty - Z_{cp_d} Z_{st} = 0. \quad (3)$$

The solutions of Eq.(3) are:

$$Z_\infty = \frac{Z_{cp_d}}{2} \pm \sqrt{Z_{cp_d} Z_{st} + \frac{Z_{cp_d}^2}{4}}. \quad (4)$$

The continuous fraction belonging to Eq.(1) converges to a limit value that represents the first solution (corresponding to the "+" sign) of Eq.(4) if the following van Vleck and Jensen theorem's conditions (Edelmann, 1966) are fulfilled:

$$\mathrm{Re}(Z_{cp_d}) > 0,\ \mathrm{Re}(Z_{st}) > 0;\ \mathrm{Im}(Z_{cp_d}) < \infty,\ \mathrm{Im}(Z_{st}) < \infty. \quad (5)$$

Therefore, the solution of Eq.(3) is:

$$Z_\infty = \frac{Z_{cp_d}}{2} + \sqrt{Z_{cp_d} Z_{st} + \frac{Z_{cp_d}^2}{4}}. \quad (6)$$

Taking into account that usually $Z_{cp_d} << Z_{st}$, Eq.(6) can be written as follows:

$$Z_{\infty} \approx \frac{Z_{cp_d}}{2} + \sqrt{Z_{cp_d} Z_{st}}. \quad (7)$$

Eq.(6) gives the impedance of an infinite section of a transmission line, extended from the fault towards one end of the line.

For an infinite line in both directions (the two sections of the line between the fault and the terminals could be considered long, Fig.2), the equivalent impedance is given by

$$\frac{1}{Z_{\infty\infty}} = \frac{1}{Z_{\infty}} + \frac{1}{Z_{st}} + \frac{1}{Z_{\infty}}, \quad (8)$$

respectively,

$$Z_{\infty\infty} = (2/Z_{\infty} + 1/Z_{st})^{-1}. \quad (9)$$

The voltage rise of the faulted tower U_0 is given by (Endrenyi, 1967):

$$U_0 = (1-\nu) I_d Z_{\infty\infty}, \quad (10)$$

where the coupling between the faulted phase conductor and the ground conductor is taken into account by Z_{cp_m}, the mutual impedance per unit length of line, and $\nu = Z_{cp_m}/Z_{cp_d}$ represents the coupling factor. I_d represents the fault current.

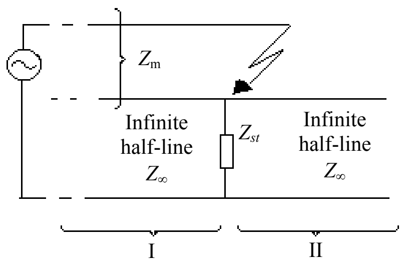

Fig.2 Full-line, infinite in both directions

Fault at the last tower

Fig.3 presents the connection of a ground wire connected to earth through transmission towers, each transmission tower having its own grounding electrode or grid, Z_{st}. When a fault appears, part of the ground fault current will get to the ground through the faulted tower, and the rest of the fault current will get diverted to the ground wire and other towers. The current I_n flowing to ground through the nth tower, counted from the terminal tower where the fault is

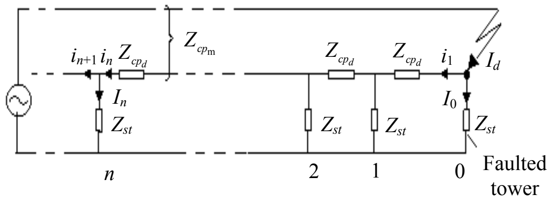

Fig.3 Fault current distribution

assumed to take place, is equal to the difference between the currents i_n and i_{n+1} (Verma and Mukhedkar, 1979):

$$I_n = i_n - i_{n+1}. \quad (11)$$

The loop equation for the nth mesh is given by

$$I_n Z_{st} - I_{n-1} Z_{st} + i_n Z_{cp_d} - \nu I_d Z_{cp_d} = 0. \quad (12)$$

Eq.(12) could be written in the form:

$$i_n = \frac{(I_{n-1} - I_n) Z_{st}}{Z_{cp_d}} + \nu I_d. \quad (13)$$

Similarly,

$$i_{n+1} = \frac{(I_n - I_{n+1}) Z_{st}}{Z_{cp_d}} + \nu I_d. \quad (14)$$

Substituting Eqs.(13) and (14) into Eq.(11), for the current in the faulted tower will yield the next equation, which is a second-order difference equation:

$$I_n \frac{Z_{cp_d}}{Z_{st}} = I_{n+1} - 2I_n + I_{n-1}. \quad (15)$$

According to (Rudenberg, 1959), the sollution of Eq.(15) is:

$$I_n = A\mathrm{e}^{\alpha n} + B\mathrm{e}^{-\alpha n}, \quad (16)$$

where the arbitrary parameters A and B could be obtained from the boundary conditions. Parameter α could be obtained by substituting Eq.(16) into Eq.(15). Because $Z_{cp_d} << Z_{st}$, it can be written:

$$\alpha \approx \sqrt{Z_{cp_d}/Z_{st}}. \quad (17)$$

Applying Eq.(11) to the ($n-1$)th tower will yield the following expression:

$$I_{n-1} = i_{n-1} - i_n. \quad (18)$$

Substituting Eqs.(11) and (18) into Eq.(12) yields the next equation with a constant term:

$$i_n \frac{Z_{cpd}}{Z_{st}} = i_{n+1} - 2i_n + i_{n-1} + \nu I_d \frac{Z_{cpd}}{Z_{st}}. \quad (19)$$

Similar to Eq.(15), the current in the ground conductor is given by:

$$i_n = a\mathrm{e}^{\alpha n} + b\mathrm{e}^{-\alpha n} + \nu I_d, \quad (20)$$

where a and b represent the arbitrary parameters.

Because of the link between currents i_n and I_n, the arbitrary parameters A, B and a, b are not independent. Substituting the solutions Eqs.(16) and (20) into Eq.(11) will yield:

$$A\mathrm{e}^{\alpha n} + B\mathrm{e}^{-\alpha n} = a\mathrm{e}^{\alpha n}(1-\mathrm{e}^{\alpha}) + b\mathrm{e}^{-\alpha n}(1-\mathrm{e}^{-\alpha}). \quad (21)$$

Because these relations are the same for every value of n, the following expressions will be obtained:

$$A = a(1-\mathrm{e}^{\alpha}), \quad (22)$$

$$B = b(1-\mathrm{e}^{-\alpha}). \quad (23)$$

The current in the ground wire will be then given by

$$i_n = A\frac{\mathrm{e}^{\alpha n}}{1-\mathrm{e}^{\alpha}} + B\frac{\mathrm{e}^{-\alpha n}}{1-\mathrm{e}^{-\alpha}} + \nu I_d. \quad (24)$$

If the line is sufficiently long so that, after some distance, the varying portion of the current decays exponentially to zero, then $A\to 0$, so in this case only B must be found from the boundary conditions (Rudenberg, 1959).

If the line cannot be considered long enough, then parameters A and B can be found from the boundary conditions. In Fig.4, R_p and R_p' represent the resistances of the grounding systems of the two stations, which are connected to the last towers, at both sides, through an extra span. In Fig.4 the impedance of this section of ground wire between the stations and the last towers was noted with Z'_{cpd}, the same at both sides. The boundary condition at the receiving end of the line is (Vintan and Buta, 2006):

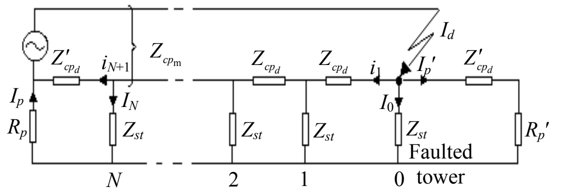

Fig.4 Fault current distribution

$$I_d = I'_p + I_0 + i_1 = i_1 + I_0 + I_0 Z_{st}/Z'_p. \quad (25)$$

At the sending end of the line, we have:

$$I_d = I_p + i_{N+1}, \quad (26)$$

$$I_N Z_{st} + I_p R_p - i_{N+1} Z'_{cpd} + \nu I_d Z'_{cpd} = 0. \quad (27)$$

Substituting I_p from Eq.(27) into Eq.(26) yields:

$$I_d\left(1+\nu\frac{Z'_{cpd}}{R_p}\right) = i_{N+1}\left(1+\frac{Z'_{cpd}}{R_p}\right) - I_N\frac{Z_{st}}{R_p}. \quad (28)$$

Substituting I_0, I_N, i_{N+1} and i_1 into Eqs.(25) and (28), according to Eqs.(16) and (24), yields a system with two linear equations (Vintan and Buta, 2006):

$$\begin{cases} I_d(1-\nu) = A\left[\dfrac{1}{1-\mathrm{e}^{\alpha}} + \dfrac{Z_{st}}{Z'_p}\right] + B\left[\dfrac{1}{1-\mathrm{e}^{-\alpha}} + \dfrac{Z_{st}}{Z'_p}\right], \\ I_d(1-\nu) = A\mathrm{e}^{\alpha N}\left[\dfrac{\mathrm{e}^{\alpha}}{1-\mathrm{e}^{\alpha}}\left(1+\dfrac{Z'_{cpd}}{R_p}\right) - \dfrac{Z_{st}}{R_p}\right] + \\ \qquad B\mathrm{e}^{-\alpha N}\left[\dfrac{\mathrm{e}^{-\alpha}}{1-\mathrm{e}^{-\alpha}}\left(1+\dfrac{Z'_{cpd}}{R_p}\right) - \dfrac{Z_{st}}{R_p}\right], \end{cases} \quad (29)$$

where $Z'_p = R'_p + Z'_{cpd}$. Eq.(29) gives

$$A = I_d(1-\nu)\frac{B_2 - B_1}{A_1B_2 - B_1A_2},\ B = I_d(1-\nu)\frac{A_1 - A_2}{A_1B_2 - B_1A_2}, \quad (30)$$

where

$$A_1 = \frac{1}{1-\mathrm{e}^{\alpha}} + \frac{Z_{st}}{Z'_p},\ B_1 = \frac{1}{1-\mathrm{e}^{-\alpha}} + \frac{Z_{st}}{Z'_p},$$

$$A_2 = \mathrm{e}^{\alpha N}\left[\frac{\mathrm{e}^{\alpha}}{1-\mathrm{e}^{\alpha}}\left(1+\frac{Z'_{cpd}}{R_p}\right) - \frac{Z_{st}}{R_p}\right],$$

$$B_2 = \mathrm{e}^{-\alpha N}\left[\frac{\mathrm{e}^{-\alpha}}{1-\mathrm{e}^{-\alpha}}\left(1+\frac{Z'_{cpd}}{R_p}\right) - \frac{Z_{st}}{R_p}\right].$$

The current in the faulted tower (obtained for n=0 in all expressions, including A_2, B_2) will be:

$$I_0 = A + B = I_d(1-\nu)\left[\frac{B_2 - B_1}{A_1B_2 - B_1A_2}(1-\mathrm{e}^{\alpha}) + \frac{A_1 - A_2}{A_1B_2 - B_1A_2}(1-\mathrm{e}^{-\alpha})\right]. \quad (31)$$

The voltage rise of the terminal tower in this case is:

$$U_0 = I_0Z_{st} = I_d(1-\nu)Z_{st}\left[\frac{B_2 - B_1}{A_1B_2 - B_1A_2}(1-\mathrm{e}^{\alpha}) + \frac{A_1 - A_2}{A_1B_2 - B_1A_2}(1-\mathrm{e}^{-\alpha})\right] = (1-\nu)I_dZ_N, \quad (32)$$

with Z_N being noted the equivalent impedance of the network looking back from the fault:

$$Z_N = Z_{st}\left[\frac{B_2 - B_1}{A_1B_2 - B_1A_2}(1-\mathrm{e}^{\alpha}) + \frac{A_1 - A_2}{A_1B_2 - B_1A_2}(1-\mathrm{e}^{-\alpha})\right]. \quad (33)$$

Fault at any tower

In Fig.5 the fault occurs at the tower number 0. There are N towers between this tower and the left station and respectively M towers between tower number 0 and the right station. The resistances of the grounding systems of the left and right stations (see Fig.5) are R_p and R_p' respectively. The impedance of the section of ground wire between one station and the first tower (tower N or M) is Z'_{cpd}, and $Z_p = R_p + Z'_{cpd}$, respectively $Z'_p = R'_p + Z'_{cpd}$. The total fault current I_d is given by the sum of the current I_d' from one side, and I_d'' from the other side of the transmission line (Dawalibi and Niles, 1984).

In this case, as the author already presented in (Vintan and Buta, 2006), considering first the left part from the faulted tower, the situation is identical to that presented in Fig.5. So, the solutions for the current in the towers, respectively in the ground wire are:

$$I_{n-s} = A_s\mathrm{e}^{\alpha n} + B_s\mathrm{e}^{-\alpha n}, \quad (34)$$

$$i_{n-s} = A_s\frac{\mathrm{e}^{\alpha n}}{1-\mathrm{e}^{\alpha}} + B_s\frac{\mathrm{e}^{-\alpha n}}{1-\mathrm{e}^{-\alpha}} + \nu I'_d. \quad (35)$$

The subscript s is used for the left part from the faulted tower. I_{n-s} represents the current in the tower number n, counted from the faulted tower to the left part of the transmission line.

In order to find the constants A_s and B_s, it is necessary to write the boundary conditions. For the faulted tower, the following formulas can be written:

$$\begin{cases} I_0Z_{st} - I_1Z_{st} - i_1Z_{cpd} + \nu Z_{cpd}I'_d = 0, \\ I_1Z_{st} - I_2Z_{st} - i_2Z_{cpd} + \nu Z_{cpd}I'_d = 0. \end{cases} \quad (36)$$

Also can be written is

$$I_1 = i_1 - i_2. \quad (37)$$

Substituting i_1 and i_2 from Eq.(36) into Eq.(37), it yields:

$$I_1(2 + Z_{cpd}/Z_{st}) = I_0 + I_2. \quad (38)$$

For the left terminal it can be written as

$$I_N + i_{N+1} = i_N, \quad (39)$$

$$I_NZ_{st} + I_pR_p - i_{N+1}Z'_{cpd} + Z'_mI'_d = 0, \quad (40)$$

$$(I_{N-1} - I_N)Z_{st} - i_NZ_{cpd} + \nu Z_{cpd}I'_d = 0, \quad (41)$$

$$i_{N+1} = I'_d - I_p, \quad (42)$$

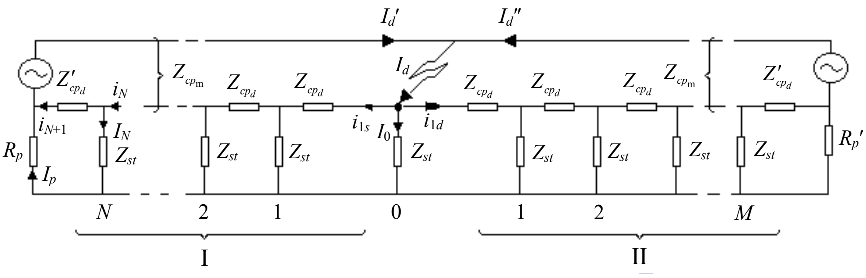

Fig.5 Ground fault current distributions

where Z_m' represents the mutual coupling between the ground wire and the faulted phase in the last span.

Replacing i_{N+1} and i_N from Eqs.(40) and (41) into Eq.(39), by taking into account Eq.(42), it yields the following expression:

$$I_N\left(1+\frac{Z_{st}}{Z_{cpd}}+\frac{Z_{st}}{Z'_{cpd}+R_p}\right)=I_{N-1}\frac{Z_{st}}{Z_{cpd}}+I'_d\left(\nu-\frac{R_p+Z'_{\mathrm{m}}}{Z'_{cpd}+R_p}\right). \quad (43)$$

Eqs.(38) and (43) represent the boundary conditions. By replacing I_1, I_2, I_N and I_{N-1} from the solutions Eqs.(34) and (35) into Eqs.(38) and (43), the expressions of A_s and B_s can be obtained (Dawalibi and Niles, 1984):

$$A_s=\frac{\mathrm{e}^{-\alpha}\left(2+\frac{Z_{cpd}}{Z_{st}}-\mathrm{e}^{-\alpha}\right)\left(\nu-\frac{R_p+Z'_{\mathrm{m}}}{R_p+Z'_{cpd}}\right)I'_d-I_0\mathrm{e}^{-\alpha N}L_s}{\mathrm{e}^{\alpha(N-1)}\left(2+\frac{Z_{cpd}}{Z_{st}}-\mathrm{e}^{-\alpha}\right)R_s-\mathrm{e}^{-\alpha(N-1)}\left(2+\frac{Z_{cpd}}{Z_{st}}-\mathrm{e}^{\alpha}\right)L_s}, \quad (44)$$

$$B_s=\frac{I_0\mathrm{e}^{\alpha N}R_s-\mathrm{e}^{\alpha}\left(2+\frac{Z_{cpd}}{Z_{st}}-\mathrm{e}^{\alpha}\right)\left(\nu-\frac{R_p+Z'_{\mathrm{m}}}{R_p+Z'_{cpd}}\right)I'_d}{\mathrm{e}^{\alpha(N-1)}\left(2+\frac{Z_{cpd}}{Z_{st}}-\mathrm{e}^{-\alpha}\right)R_s-\mathrm{e}^{-\alpha(N-1)}\left(2+\frac{Z_{cpd}}{Z_{st}}-\mathrm{e}^{\alpha}\right)L_s}, \quad (45)$$

where

$$L_s=1+(1-\mathrm{e}^{\alpha})\frac{Z_{st}}{Z_{cpd}}+\frac{Z_{st}}{R_p+Z'_{cpd}},$$

$$R_s=1+(1-\mathrm{e}^{-\alpha})\frac{Z_{st}}{Z_{cpd}}+\frac{Z_{st}}{R_p+Z'_{cpd}}.$$

Similar expressions are obtained for the currents from the right part of the faulted tower:

$$\begin{cases} I_{n-d}=A_d\mathrm{e}^{\alpha n}+B_d\mathrm{e}^{-\alpha n}, \\ i_{n-d}=\frac{A_d}{1-\mathrm{e}^{\alpha}}\mathrm{e}^{\alpha n}+\frac{B_d}{1-\mathrm{e}^{-\alpha}}\mathrm{e}^{-\alpha n}+\nu I''_d, \end{cases} \quad (46)$$

where

$$A_d=\frac{\mathrm{e}^{-\alpha}\left(2+\frac{Z_{cpd}}{Z_{st}}-\mathrm{e}^{-\alpha}\right)\left(\nu-\frac{R'_p+Z'_{\mathrm{m}}}{R'_p+Z'_{cpd}}\right)I''_d-I_0\mathrm{e}^{-\alpha M}L_d}{\mathrm{e}^{\alpha(M-1)}\left(2+\frac{Z_{cpd}}{Z_{st}}-\mathrm{e}^{-\alpha}\right)R_d-\mathrm{e}^{-\alpha(M-1)}\left(2+\frac{Z_{cpd}}{Z_{st}}-\mathrm{e}^{\alpha}\right)L_d}, \quad (47)$$

$$B_d=\frac{I_0\mathrm{e}^{\alpha M}R_d-\mathrm{e}^{\alpha}\left(2+\frac{Z_{cpd}}{Z_{st}}-\mathrm{e}^{\alpha}\right)\left(\nu-\frac{R'_p+Z'_{\mathrm{m}}}{R'_p+Z'_{cpd}}\right)I''_d}{\mathrm{e}^{\alpha(M-1)}\left(2+\frac{Z_{cpd}}{Z_{st}}-\mathrm{e}^{-\alpha}\right)R_d-\mathrm{e}^{-\alpha(M-1)}\left(2+\frac{Z_{cpd}}{Z_{st}}-\mathrm{e}^{\alpha}\right)L_d}, \quad (48)$$

where

$$L_d=1+(1-\mathrm{e}^{\alpha})\frac{Z_{st}}{Z_{cpd}}+\frac{Z_{st}}{R'_p+Z'_{cpd}},$$

$$R_d=1+(1-\mathrm{e}^{-\alpha})\frac{Z_{st}}{Z_{cpd}}+\frac{Z_{st}}{R'_p+Z'_{cpd}}.$$

At the faulted tower the total ground fault is (Fig.5):

$$I_d=I_0-i_{1s}-i_{1d}. \quad (49)$$

The current in the faulted tower I_0 becomes:

$$I_0=\frac{(1-\nu)I_d+\left(\frac{W_s}{T_s}I'_d+\frac{W_d}{T_d}I''_d\right)\left(\frac{V_1}{1-\mathrm{e}^{-\alpha}}-\frac{V_2}{1-\mathrm{e}^{\alpha}}\right)}{1-\frac{\mathrm{e}^{\alpha}}{1-\mathrm{e}^{\alpha}}\left(\frac{L_s\mathrm{e}^{-\alpha N}}{T_s}+\frac{L_d\mathrm{e}^{-\alpha M}}{T_d}\right)+\frac{\mathrm{e}^{-\alpha}}{1-\mathrm{e}^{-\alpha}}\left(\frac{R_s\mathrm{e}^{\alpha N}}{T_s}+\frac{R_d\mathrm{e}^{\alpha M}}{T_d}\right)}, \quad (50)$$

where

$$W_s=\nu-\frac{R_p+Z'_{\mathrm{m}}}{R_p+Z'_{cpd}},\quad W_d=\nu-\frac{R'_p+Z'_{\mathrm{m}}}{R'_p+Z'_{cpd}},$$

$$V_1=2+\frac{Z_{cpd}}{Z_{st}}-\mathrm{e}^{\alpha},\quad V_2=2+\frac{Z_{cpd}}{Z_{st}}-\mathrm{e}^{-\alpha},$$

$$T_s=\mathrm{e}^{\alpha(N-1)}\left(2+\frac{Z_{cpd}}{Z_{st}}-\mathrm{e}^{-\alpha}\right)R_s-\mathrm{e}^{-\alpha(N-1)}\left(2+\frac{Z_{cpd}}{Z_{st}}-\mathrm{e}^{-\alpha}\right)L_s,$$

$$T_d=\mathrm{e}^{\alpha(M-1)}\left(2+\frac{Z_{cpd}}{Z_{st}}-\mathrm{e}^{-\alpha}\right)R_d-\mathrm{e}^{-\alpha(M-1)}\left(2+\frac{Z_{cpd}}{Z_{st}}-\mathrm{e}^{-\alpha}\right)L_d.$$

The voltage rise of the terminal tower is:

$$U=I_0Z_{st}=\frac{(1-\nu)I_d+\left(\frac{W_s}{T_s}I'_d+\frac{W_d}{T_d}I''_d\right)\left(\frac{V_1}{1-\mathrm{e}^{-\alpha}}-\frac{V_2}{1-\mathrm{e}^{\alpha}}\right)}{1-\frac{\mathrm{e}^{\alpha}}{1-\mathrm{e}^{\alpha}}\left(\frac{L_s\mathrm{e}^{-\alpha N}}{T_s}+\frac{L_d\mathrm{e}^{-\alpha M}}{T_d}\right)+\frac{\mathrm{e}^{-\alpha}}{1-\mathrm{e}^{-\alpha}}\left(\frac{R_s\mathrm{e}^{\alpha N}}{T_s}+\frac{R_d\mathrm{e}^{\alpha M}}{T_d}\right)}Z_{st}. \quad (51)$$

RESULTS

In order to illustrate the theoretical approach outlined in the previous section, we are considering that the line which connects two stations is a 110-kV transmission line with aluminium-steel 185/32 mm^2 and one aluminium-steel ground wire 95/55 mm^2 (Fig.6) (ICEMENERG, 1993). Line impedances per span are determined based on the following assumptions: average length of the span is 250 m; the resistance per unit length of ground wire is 0.3 Ω/km and its diameter is 16 mm. Ground wire impedance per span Z_{cp_d} and the mutual impedance Z_m between the ground wire and the faulted phase are calculated for different values of the soil resistivity ρ with formulae based on Carson's theory of the ground return path (Carson, 1926). Impedance Z_m is calculated only in relation to the faulted phase conductor, because it could not be assumed that a line section of a few spans is transposed. The fault was assumed to occur on the phase which is the furthest from the ground conductors, because the lowest coupling between the phase and ground wire will produce the highest tower voltage.

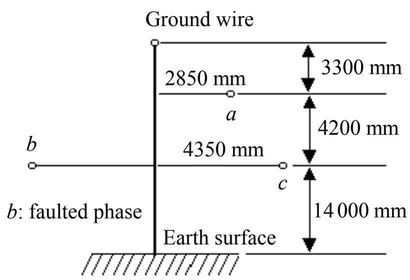

Fig.6 Disposition of line conductors

Fig.7 presents the values of the impedance of the infinite half-line, as a function of the towers impedances, for different values of the ground wire.

Fig.8 presents the values of the equivalent impedance of the line, composed of two infinite half-line, for different values of the towers impedances.

Fig.9 shows the values for the impedance of finite line, in case of a fault at the last tower of the line.

In order to see the effect of the mutual coupling between the faulted phase and the ground wire, the term $(1-\nu)Z_N$ is represented in Fig.10.

Fig.11 presents the values for the finite line impedance calculated with the proposed Eq.(33) and with the Endrenyi formula (Endrenyi, 1967). As can be observed, the two formulae are generating quite identical values.

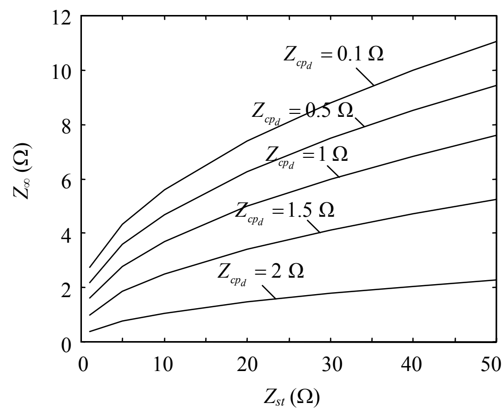

Fig.7 The infinite line impedance as a function of the towers impedances

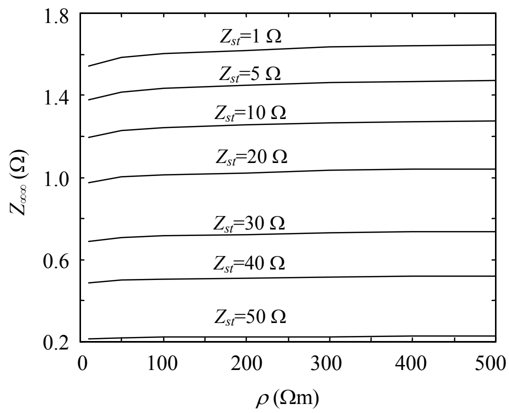

Fig.8 The equivalent impedance of the line infinite in both directions

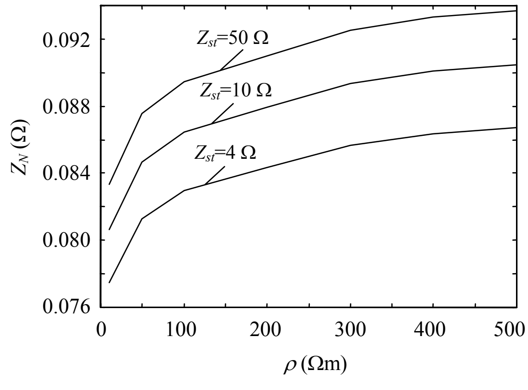

Fig.9 Finite line impedance

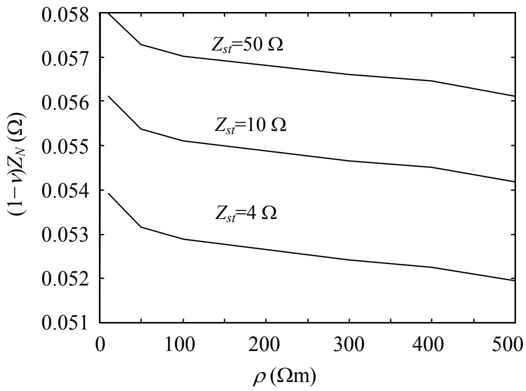

Fig.10 The finite line impedance taking into account the mutual coupling between the faulted phase and the ground wire

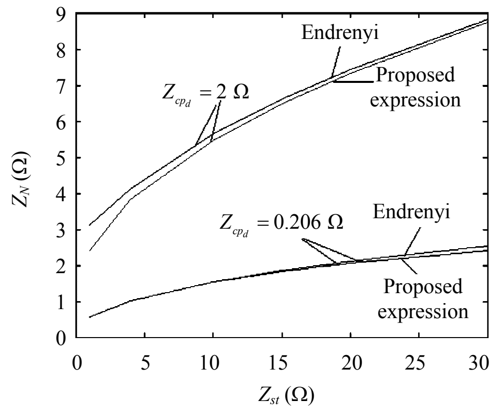

Fig.11 The finite line impedance

CONCLUSION

This paper presents an improved analytical method for assessing the effects of a ground fault at any tower of a transmission line. The method is applied to three-phase power network, with overhead transmission lines.

During ground faults on transmission lines, a number of towers near the fault acquire high potentials to ground. Since during a ground fault the maximum voltage will appear at the tower nearest to the fault, attention in this study was focused on that tower. A phase-to-ground fault occurring on a transmission line divides the line into two sections, each extending from the fault towards one end of the line. This paper describes an analytical method to determine the voltage rise of the faulted tower for those sections of the line.

Based on the earlier approach of (Endrenyi, 1967), it was shown that using the continuous fractions theory (Edelmann, 1966), the impedance of the infinite half-line will have the same expression as that presented by Endrenyi. The author also develops a new original analytical method to evaluate the impedance of the finite line.

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