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Ground Fault Current Distribution on Overhead Transmission Lines

Maria Vintan and Adrian Buta

Abstract: When a ground fault occurs on an overhead transmission line in a power network with grounded neutral, the fault current returns to the grounded neutral through the tower structures, ground return paths and ground wires. This paper presents an analytical method in order to evaluate the ground fault current distribution in an effectively grounded power network. The effect of soil resistivity, ground resistance of towers and power line configuration, on the magnitude of return currents, has been examined.

Keywords: Overhead transmission line, fault current distribution, power network.

1 Introduction

High-voltage systems have an effectively grounded neutral. When a ground fault occurs on an overhead transmission line, in a power network with grounded neutral, the fault current returns to the grounded neutral through the tower structures, ground return paths and ground wires [1],[2]. In this paper, based on Kirchhoff's theorems, an analytical method in order to determine the ground fault current distribution in effectively grounded power network is presented. It is possible to find the values of the currents in towers, ground wire and the currents who return to the stations [3],[4],[5],[6]. The approach used in this paper, based on method presented by Rudenberg [7] and Verma [8], can be applied to short lines, respectively in case where are only few spans between the feeding station and the faulted tower.

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A phase-to-ground fault that appears on a phase of a transmission line, divides the line into two sections, each extending from the fault towards one end of the line. Depending of the number of towers between the faulted tower and the stations, respectively of the distance between the towers, these two sections of the line may be considered infinite, in which case the ground fault current distribution is independent on the termination of the network, otherwise, they must be regarded as finite, in which case the ground fault current distribution may depend greatly on the termination of the network [5],[6].

In this paper it will be considered the case when the fault appears to the last tower of the transmission line. Then, it will be considered that the fault appears at any tower of the transmission line, the two sections of the line are finite and it is assumed that the fault is fed from both directions [9],[10].

The calculation method is based on the following assumptions: impedances are considered as lumped parameters in each span of the transmission line, line capacitances are neglected; the contact resistance between the tower and the ground wire, and the tower resistance between the ground wire and the faulted phase are neglected; the network is considered linear in the sinusoidal steady-state and only the power frequency is considered.

2 Ground fault current distribution

We presented two cases: first case is the fault at the terminal tower of the line and the second case the fault at any tower of the line.

Case 1. Figure 1 presents the connection of a ground wire connected to earth through transmission towers, each transmission tower having its own grounding electrode or grid, Z_{st}. It is assumed that all the transmission towers have the same ground impedance and the distance between towers is long enough to avoid the influence between there grounding electrodes. The self-impedance of the ground wire connected between two grounded towers, called the self-impedance per span, it is Z_{cp_d} . Considering the same distance I_d between two consecutive towers and that Z_{cp_d} is the same for every span, then $Z_{cp} = I_d Z_{cp_d}$, where represent the self-impedance of the ground wire in Ω/km . Z_{cp_m} represents the mutual-impedance between the ground wire and the faulted phase conductor, per span.

When a fault appears, part of the ground fault current will get to the ground through the faulted tower, and the rest of the fault current will get diverted to the ground wire and other towers. The current I_n flowing to ground through the n-th tower, counted from the terminal tower where the fault is assumed to take place, is

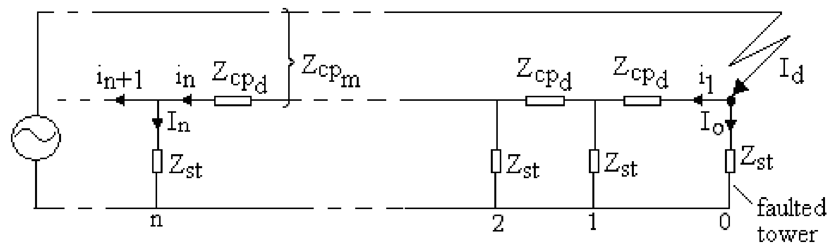

Fig. 1. Fault current distribution, case 1.

equal with the difference between the currents i_n and i_{n+1} [7],[8]:

$$I_n = i_n - i_{n+1} \tag{1}$$

The next relation gives the loop equation for the n-th mesh:

$$I_n Z_{st} - I_{n-1} Z_{st} + i_n Z_{cp_d} - \nu I_d Z_{cp_d} = 0 \tag{2}$$

In equation (2) $\nu = Z_{cp_m}/Z_{cp_d}$ represents the coupling factor between the overhead phase and ground wire and I_d represents the fault current. The equation (2) could be write in the next form:

$$i_n = (I_{n-1} - I_n)\frac{Z_{st}}{Z_{cp_d}} + \nu I_d \tag{3}$$

and similarly

$$i_{n+1} = (I_n - I_{n+1})\frac{Z_{st}}{Z_{cp_d}} + \nu I_d \tag{4}$$

Substituting equations (3) and (4) in equation (1), for the current in the faulted tower will be obtained the next equation, which is a second order difference equation:

$$I_n \frac{Z_{cp_d}}{Z_{st}} = I_{n+1} - 2I_n + I_{n-1} \tag{5}$$

According to [7], the solution of this equation is:

$$I_n = Ae^{\alpha n} + Be^{-\alpha n} \tag{6}$$

According with the solution (6) that contains the arbitrary parameters A and B, the current flowing to ground through the successive towers, has an exponential variation. The arbitrary parameters A and B will be obtained later, from the boundary conditions.

Parameter α in the solution (6) could be obtained by substituting the solution (6) in equation (5). For this purpose, n is substituting with $(n+1)$, respectively with $(n-1)$ in equation (6):

$$I_{n+1} = Ae^{\alpha(n+1)} + Be^{-\alpha(n+1)} \tag{7}$$

$$I_{n-1} = Ae^{\alpha(n-1)} + Be^{-\alpha(n-1)} \tag{8}$$

The equation (5) became:

$$\frac{Z_{cp_d}}{Z_{st}} = e^{\alpha} + e^{-\alpha} - 2 = 2(\sinh\frac{a}{2})^2 \tag{9}$$

Because $Z_{cp_d} \ll Z_{st}$, it can be written:

$$\alpha = \sqrt{\frac{Z_{cp_d}}{Z_{st}}} \tag{10}$$

By applying equation (1) to the $(n-1)$ tower, it will be obtained the following expression:

$$I_{n-1} = i_{n-1} - i_n \tag{11}$$

By substituting the equations (1) and (11) in (2), it will be obtained the next equation with a constant term:

$$i_n \frac{Z_{cp_d}}{Z_{st}} = i_{n+1} - 2i_n + i_{n-1} + \nu I_d \frac{Z_{cp_d}}{Z_{st}} \tag{12}$$

Similar with equation (5), the current in the ground conductor is given by the next solution:

$$i_n = ae^{\alpha n} + be^{-\alpha n} + \nu I_d \tag{13}$$

a, b represents the arbitrary parameters.

Because of the link between currents i_n and I_n, the arbitrary parameters A, B and a, b are not independent. By substituting the solutions (6) and (13) in equation (1), it will be obtained:

$$Ae^{\alpha n} + Be^{-\alpha n} = ae^{\alpha n}(1 - e^{\alpha}) + be^{-\alpha n}(1 - e^{-\alpha}) \tag{14}$$

Because these relations are the same for every value of n, it will be obtained the next expressions:

$$A = a(1 - e^{\alpha}) \tag{15}$$

$$B = b(1 - e^{-\alpha}) \tag{16}$$

The current in the ground wire will be then given by the following expression:

$$i_n = A\left(\frac{e^{\alpha n}}{1-e^{\alpha}}\right) + B\left(\frac{e^{-\alpha n}}{1-e^{-\alpha}}\right) + \nu I_d \tag{17}$$

The boundary condition (condition for $n = 0$) at the terminal tower of Figure 1, which means that the fault current is given by the sum between the current in the faulted tower and the current in the first span of the ground wire, is:

$$I_d = I_0 + i_1 \tag{18}$$

Long Line. If the line is sufficiently long so that, after some distance, the varying portion of the current exponentially decays to zero, $A \rightarrow 0$, and according to (6) and (17):

$$I_n = Be^{-\alpha n} \tag{19}$$

$$i_n = B\left(\frac{e^{-\alpha n}}{1-e^{-\alpha}}\right) + \nu I_d \tag{20}$$

Substituting these expressions in (18), with $n = 0$ for I_n and $n = 1$ for i_n, it will be obtained:

$$I_d = B + B\left(\frac{e^{-\alpha}}{1-e^{-\alpha}}\right) + \nu I_d = \left(\frac{B}{1-e^{-\alpha}}\right) + \nu I_d \tag{21}$$

For B it will be obtained the next expression:

$$B = (1-\nu)(1-e^{-\alpha})I_d = \frac{(1-\nu)(2\tanh\frac{\alpha}{2})}{(1+\tanh\frac{\alpha}{2}))I_d} \tag{22}$$

The current in the faulted tower will get the expression:

$$I_0 = B = \frac{(1-\nu)(2\tanh\frac{\alpha}{2})}{(1+\tanh\frac{\alpha}{2}))I_d} \tag{23}$$

The current in the first span, counted from the faulted tower, will be:

$$i_1 = I_d - I_0 = I_d[e^{-\alpha} + \nu(1-e^{-\alpha})] \tag{24}$$

The voltage rise at the terminal tower is:

$$\begin{aligned} U_0 = Z_{st} I_0 &= Z_{st}(1-\nu)(1-e^{-\alpha})I_d \\ &= \frac{(1-\nu)(2\tanh\frac{\alpha}{2})}{(1+\tanh\frac{\alpha}{2})} Z_{st} I_d \\ &= (1-\nu) Z I_d \end{aligned} \tag{25}$$

where Z represents the equivalent impedance of the network looking back from the fault.

Usually, the terminal tower is connected, through an extra span Z'_{cp_d}, to the station-grounding grid (Figure 2). Consequently, a resistance representing the grounding system of the station resistance must close the ladder network representing such a line. In this case, a part of the total ground fault current will flow through the station ground resistance R'_p. In order to use the previous results, it is enough to replace the current I_d with $I'_d = I_d - I'_p$, and thus the value of the current in the faulted tower will be [8]:

$$I_0 = (1-\nu)(1-e^{-\alpha})I'_d \tag{26}$$

Noting the sum between Z'_{cp_d} and R'_p with $Z'_p = R'_p + Z'_{cp_d}$, the current I'_p through the station grounding grid resistance is given by the next expression:

$$I'_p = \frac{I_d Z}{Z'_p + Z} \tag{27}$$

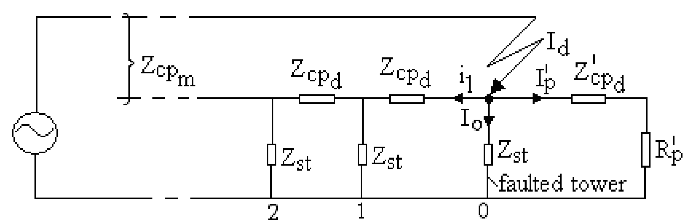

Fig. 2. Fault current distributions.

In case the values of Z'_p and Z are known, I'_p can be found out from (27) above. I'_d is given by the next expression:

$$I'_d = I_d - I'_p = I_0 + i_1 \tag{28}$$

Now, it will be found the n-th tower, as counted from the terminal tower, where the current gets reduced to 1% then that traversing the terminal tower. From equation (19):

$$Be^{-\alpha n} = \frac{1}{100B} \Rightarrow n = \frac{1}{\alpha \ln 100} \approx 4,6\sqrt{\frac{Z_{st}}{Z_{cp_d}}} \tag{29}$$

For $\frac{Z_{cp_d}}{Z_{st}} = 0,03$, will get $n = 26,5$, so it takes at least 26 towers from the fault to get a current reduced to 1% then that traversing the terminal tower. Taking into account this consideration, it is possible to see if could be considered $A = 0$ in the expressions of the currents in ground wire and towers. The number of the towers should be at least equal with the number given by expression (29).

Short line. If the line cannot be considered long enough regarding to the expression (29), then parameters A and B will be found from the boundary conditions. With R_p, respectively $R_p^{'}$, are noted the resistances of the grounding systems of the two stations (Figure 3), which doesn't include the grounding effects of the ground wire of the considered line. The stations are connected to the terminal towers, at the both sides, through an extra span $Z_{cp_d}^{'}$, and the sum between $Z_{cp_d}^{'}$ and grounding system of the stations resistances was noted with $Z_p = R_p + Z_{cp_d}^{'}$, respectively $Z_p^{'} = R_p^{'} + Z_{cp_d}^{'}$. The boundary condition at the receiving end of the line is:

$$I_d = I_p^{'} + I_0 + i_1 = i_1 + I_0 + I_0 \frac{Z_{st}}{Z_p^{\prime}} \tag{30}$$

At the sending end of the line we have:

$$I_d = I_p + i_{N+1} \tag{31}$$

$$I_N Z_{st} + I_p R_p - i_{N+1} Z_{cp_d}^{'} + \nu I_d Z_{cp_d}^{'} = 0 \tag{32}$$

Substituting I_p from (32) to (31), we'll get:

$$I_d \left(1 + \nu \frac{Z_{cp_d}^{'}}{R_p}\right) = i_{N+1}(1 + Z_{cp_d}^{'} R_p) + \frac{I_N Z_{st}}{R_p} \tag{33}$$

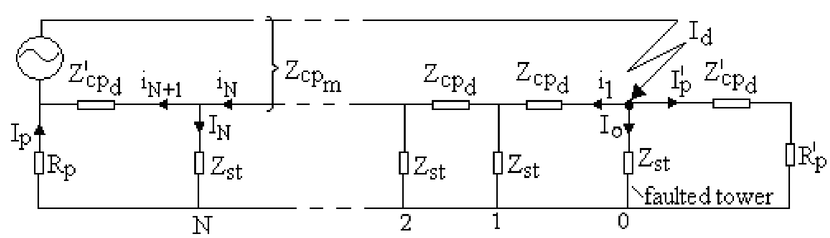

Fig. 3. Fault current distributions.

Substituting I_0, I_N, i_{N+1} and i_1 into (30) and (33), according to (6) and (17), it will be obtained a system with two linear equations:

$$\begin{cases} I_d(1-\nu) = A\left[\frac{1}{1-e^{\alpha}} + \frac{Z_{st}}{Z_p^{\prime}}\right] + B\left[\frac{1}{1-e^{-\alpha}} + \frac{Z_{st}}{Z_p^{\prime}}\right] \\ I_d(1-\nu) = Ae^{\alpha N}\left[\frac{e^{\alpha}}{1-e^{\alpha}}(1 + \frac{Z_{cp_d}^{'}}{R_p}) - \frac{Z_{st}}{R_p}\right] + Be^{-\alpha N}\left[\frac{e^{-\alpha}}{1-e^{-\alpha}}(1 + \frac{Z_{cp_d}^{'}}{R_p}) - \frac{Z_{st}}{R_p}\right] \end{cases} \tag{34}$$

(34) gives:

$$A = I_d(1-\nu)\frac{B_2 - B_1}{A_1B_2 - B_1A_2}, B = I_d(1-\nu)\frac{A_1 - A_2}{A_1B_2 - B_1A_2} \tag{35}$$

where: A_1, B_1, A_2, B_2 are:

$$A_1 = \frac{1}{1-e^{\alpha}} + \frac{Z_{st}}{Z_p^{'}}, B_1 = \frac{1}{1-e^{-\alpha}} + \frac{Z_{st}}{Z_p^{'}}$$

$$A_2 = e^{\alpha N}\left[\frac{e^{\alpha}}{1-e^{\alpha}}\left(1+\frac{Z_{cp_d}^{'}}{R_p}\right) - \frac{Z_{st}}{R_p}\right], B_2 = e^{-\alpha N}\left[\frac{e^{-\alpha}}{1-e^{-\alpha}}\left(1+\frac{Z_{cp_d}^{'}}{R_p}\right) - \frac{Z_{st}}{R_p}\right]$$

The current in the faulted tower (obtained for $n = 0$ in all expressions, including A_2, B_2) will be:

$$I_0 = A + B = I_d(1-\nu)\left[\frac{B_2 - B_1}{A_1B_2 - B_1A_2}(1-e^{\alpha}) + \frac{A_1 - A_2}{A_1B_2 - B_1A_2}(1-e^{-\alpha})\right] \tag{36}$$

The voltage rise of the terminal tower in this case is:

$$\begin{aligned} U_0 &= I_0 Z_{st} \\ &= I_d(1-\nu)Z_{st}\left[\frac{B_2 - B_1}{A_1B_2 - B_1A_2}(1-e^{\alpha}) + \frac{A_1 - A_2}{A_1B_2 - B_1A_2}(1-e^{-\alpha})\right] \\ &= (1-\nu)I_d Z_N \end{aligned} \tag{37}$$

With Z_N was noted the equivalent impedance of the network looking back from the fault in this case:

$$Z_N = Z_{st}\left[\frac{B_2 - B_1}{A_1B_2 - B_1A_2}(1-e^{\alpha}) + \frac{A_1 - A_2}{A_1B_2 - B_1A_2}(1-e^{-\alpha})\right] \tag{38}$$

Case 2. In Figure 4 the fault occurs at the tower number 0. There are N towers between this tower and the left station and respectively M towers between tower number 0 and the right station. The resistances of the grounding systems of the left and right stations (Figure 4) are R_p, respectively $R_p^{'}$. The impedance of the section of ground wire between one station and the first tower (tower N or M) is $Z_{cp_d}^{'}$, and $Z_p = R_p + Z_{cp_d}^{'}$, respectively $Z_p^{'} = R_p^{'} + Z_{cp_d}^{'}$. The total fault current I_d is given by the sum between the current $I_d^{'}$ from one side, and $I_d^{"}$ from the other side of the transmission line.

In this case, considering first the left part from the faulted tower, the situation is identically with that presented in Figure 1. So it could be write the same equations

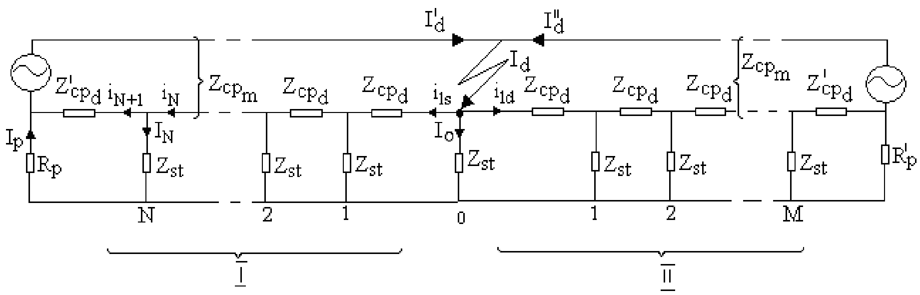

Fig. 4. Ground fault current distributions.

(1 - 17), and the solutions for the current in the towers, respectively in the ground wire are:

$$I_{n-s} = A_s e^{\alpha n} + B_s e^{-\alpha n} \tag{39}$$

$$i_{n-s} = A_s \frac{e^{\alpha n}}{1-e^{\alpha}} + B_s \frac{e^{-\alpha n}}{1-e^{-\alpha}} + \nu I_d^{'} \tag{40}$$

The subscript s is used for the left part from the faulted tower. I_{n-s} represents the current in the tower number n, counted from the faulted tower to the left part of the transmission line.

In order to find the constants A_s, B_s, it is necessary to write the boundary conditions. For the faulted tower, the following formulas can be written:

$$\begin{cases} I_0 Z_{st} - I_1 Z_{st} - i_1 Z_{cp_d} + \nu Z_{cp_d} I_d^{'} = 0 \\ I_1 Z_{st} - I_2 Z_{st} - i_2 Z_{cp_d} + \nu Z_{cp_d} I_d^{'} = 0 \end{cases} \tag{41}$$

Also can be written the next expression:

$$I_1 = i_1 - i_2 \tag{42}$$

Substituting i_1 and i_2 from equations (41) in equation (42), it will be obtained:

$$I_1 \left(2 + \frac{Z_{cp_d}}{Z_{st}} \right) = I_0 + I_2 \tag{43}$$

For the left terminal it can be written:

$$I_N + i_{N+1} = i_N \tag{44}$$

$$I_N Z_{st} + I_p R_p - i_{N+1} Z_{cp_d}^{'} + I_d^{'} Z_m^{'} = 0 \tag{45}$$

$$(I_{N-1} - I_N) Z_{st} - i_N Z_{cp_d} + \nu Z_{cp_d} I_d^{'} = 0 \tag{46}$$

$$i_{N+1} = I_d^{'} - I_p \tag{47}$$

where $Z_m^{'}$ represents the mutual coupling between the ground wire and the faulted phase, in the last span.

Replacing i_{N+1} and i_N from (45), (46) in (44), by taking into account (47), it will be obtained the following expression:

$$I_N\left(1+\frac{Z_{st}}{Z_{cp_d}}+\frac{Z_{st}}{Z_{cp_d}^{'}+R_p}\right) = I_{N-1}\frac{Z_{st}}{Z_{cp_d}} + I_d^{'}\left(\nu - \frac{R_p+Z_m^{'}}{Z_{cp_d}^{'}+R_p}\right) \tag{48}$$

Expressions (43) and (48) represents the boundary conditions. By replacing I_1, I_2, I_N and I_{N-1} from the solutions (39) and (40) in (43) and (48), it will be obtained a two system equations from which results A_s and B_s:

$$A_s = \frac{e^{-\alpha}\left(2+\frac{Z_{cp_d}}{Z_{st}}-e^{-\alpha}\right)\left(\nu-\frac{R_p+Z_m^{'}}{R_p+Z_{cp_d}^{'}}\right)I_d^{'} - I_0 e^{-\alpha N}L_s}{e^{\alpha(N-1)}\left(2+\frac{Z_{cp_d}}{Z_{st}}-e^{-\alpha}\right)R_s - e^{-\alpha(N-1)}\left(2+\frac{Z_{cp_d}}{Z_{st}}-e^{\alpha}\right)L_s} \tag{49}$$

$$B_s = \frac{I_0 e^{\alpha N}R_s - e^{\alpha}\left(2+\frac{Z_{cp_d}}{Z_{st}}-e^{\alpha}\right)\left(\nu-\frac{R_p+Z_m^{'}}{R_p+Z_{cp_d}^{'}}\right)I_d^{'}}{e^{\alpha(N-1)}\left(2+\frac{Z_{cp_d}}{Z_{st}}-e^{-\alpha}\right)R_s - e^{-\alpha(N-1)}\left(2+\frac{Z_{cp_d}}{Z_{st}}-e^{\alpha}\right)L_s} \tag{50}$$

where:

$$L_s = 1+(1-e^{\alpha})\frac{Z_{st}}{Z_{cp_d}}+\frac{Z_{st}}{R_p+Z_{cp_d}^{'}}, R_s = 1+(1-e^{-}\alpha)\frac{Z_{st}}{Z_{cp_d}}+\frac{Z_{st}}{R_p+Z_{cp_d}^{'}}$$

Similar expressions are obtained for the currents from the right part of the faulted tower:

$$\begin{cases} I_{n-d} = A_d e^{\alpha n} + B_d e^{-\alpha n} \\ i_{n-d} = \frac{A_d}{1-e^{\alpha}}e^{\alpha n} + \frac{B_d}{1-e^{-\alpha}}e^{-\alpha n} + \nu I_d^{”} \end{cases} \tag{51}$$

The constants A_d and B_d are given by the next expressions:

$$A_d = \frac{e^{-\alpha}\left(2+\frac{Z_{cp_d}}{Z_{st}}-e^{-\alpha}\right)\left(\nu-\frac{R_p^{'}+Z_m^{'}}{R_p^{'}+Z_{cp_d}^{'}}\right)I_d^{”} - I_0 e^{-\alpha M}L_d}{e^{\alpha(M-1)}\left(2+\frac{Z_{cp_d}}{Z_{st}}-e^{-\alpha}\right)R_d - e^{-\alpha(M-1)}\left(2+\frac{Z_{cp_d}}{Z_{st}}-e^{\alpha}\right)L_d} \tag{52}$$

$$B_d = \frac{I_0 e^{\alpha M}R_d - e^{\alpha}\left(2+\frac{Z_{cp_d}}{Z_{st}}-e^{\alpha}\right)\left(\nu-\frac{R_p^{'}+Z_m^{'}}{R_p^{'}+Z_{cp_d}^{'}}\right)I_d^{”}}{e^{\alpha(M-1)}\left(2+\frac{Z_{cp_d}}{Z_{st}}-e^{-\alpha}\right)R_d - e^{-\alpha(M-1)}\left(2+\frac{Z_{cp_d}}{Z_{st}}-e^{\alpha}\right)L_d} \tag{53}$$

where:

$$L_d = 1 + (1 - e^{\alpha})\frac{Z_{st}}{Z_{cp_d}} + \frac{Z_{st}}{R_p^{'} + Z_{cp_d}^{'}}, R_d = 1 + (1 - e^{-\alpha})\frac{Z_{st}}{Z_{cp_d}} + \frac{Z_{st}}{R_p^{'} + Z_{cp_d}^{'}}$$

At the faulted tower the total ground fault is (figure 4):

$$I_d = I_0 - i_{1s} - i_{1d} \tag{54}$$

The current in the faulted tower I_0 became:

$$I_0 = \frac{(1-\nu)I_d + \left(\frac{W_s}{T_s}I_d^{'} + \frac{W_d}{T_d}I_d^{"}\right)\left(\frac{V_1}{1-e^{-\alpha}} - \frac{V_2}{1-e^{\alpha}}\right)}{1 - \frac{e^{\alpha}}{1-e^{\alpha}}\left(\frac{L_s e^{-\alpha N}}{T_s} + \frac{L_d e^{-\alpha M}}{T_d}\right) + \frac{e^{-\alpha}}{1-e^{-\alpha}}\left(\frac{R_s e^{\alpha N}}{T_s} + \frac{R_d e^{\alpha M}}{T_d}\right)} \tag{55}$$

where:

$$W_s = \nu - \frac{R_p + Z_m^{'}}{R_p + Z_{cp_d}^{'}}, W_d = \nu - \frac{R_p^{'} + Z_m^{'}}{R_p^{'} + Z_{cp_d}^{'}}$$

$$V_1 = 2 + \frac{Z_{cp_d}}{Z_{st}} - e^{\alpha}, V_2 = 2 + \frac{Z_{cp_d}}{Z_{st}} - e^{-\alpha}$$

$$T_s = e^{\alpha(N-1)}\left(2 + \frac{Z_{cp_d}}{Z_{st}} - e^{-\alpha}\right)R_s - e^{-\alpha(N-1)}\left(2 + \frac{Z_{cp_d}}{Z_{st}} - e^{-\alpha}\right)L_s,$$

$$T_d = e^{\alpha(M-1)}\left(2 + \frac{Z_{cp_d}}{Z_{st}} - e^{-\alpha}\right)R_d - e^{-\alpha(M-1)}\left(2 + \frac{Z_{cp_d}}{Z_{st}} - e^{-\alpha}\right)L_d$$

3 Numerical Results

In order to illustrate the theoretical approach outlined in section above, we are considering that the line that connects two stations is an 110 kV transmission line with aluminium-steel 185/32mm^2 and one aluminium-steel ground wire 95/55mm^2 (Figure 5). Line impedances per one span are determined on the bases of the following assumptions: average length of the span is 250m; the resistances per unit length of ground wire is $0.3\Omega/\text{km}$ and it's diameter is $16mm$. Ground wire impedance per one span Z_{cp_d} and the mutual impedance Z_m between the ground wire and the faulted phase are calculated for different values of the soil resistivity ρ with formulas based on Carson's theory of the ground return path, given in the Appendix 1. Impedance Z_m is calculated only in relation to the faulted phase conductor, because it could not be assumed that a line section of a few spans are transposed. The fault was assumed to occur on the phase, which is the furthest from the ground wire,

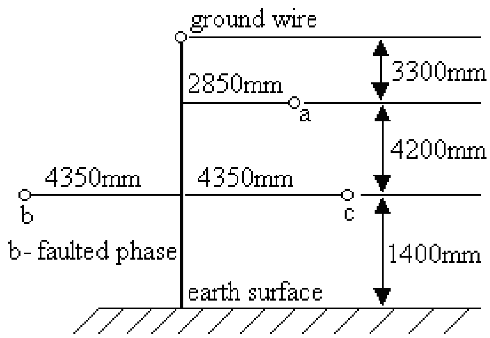

Fig. 5. Disposition of line conductors.

because the lowest coupling between the phase and ground wire will produce the highest tower voltage.

Figure 6 shows the values for the impedance of finite line in case of a fault at the last tower of the line, as a function of the of the soil resistivity and for different values of towers impedances. The values are calculated with the proposed expression (38).

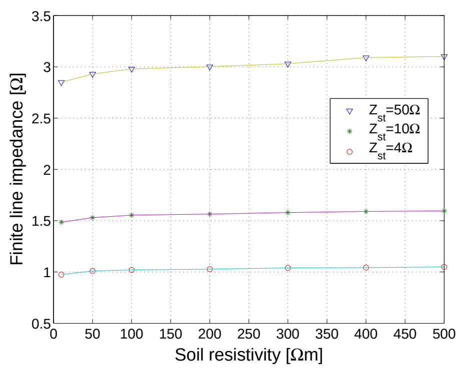

Fig. 6. Impedance of finite line for different values of soil resistivity.

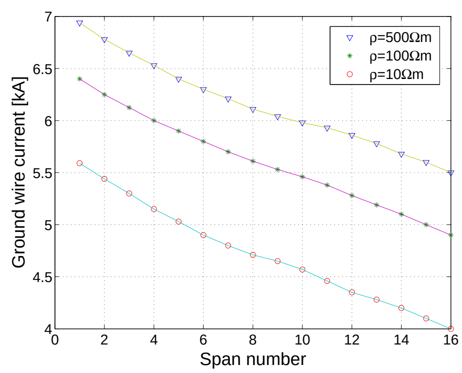

Fig. 7. Ground wire current for different values of ground wire and mutual impedance in case of the fault at the last tower. $Z_{st} = 10\Omega$.

Figure 7 shows the currents flowing in the ground wire in case of a fault at the last tower at the line. It was assumed that the line has 15 towers. The values of ground wire and mutual impedance between the faulted phase and the ground wire given in Table 1, were calculated for different values of the soil resistivity. For those values and for that number of tower, according with the expression (29), the line should be considered a short line.

In Table 1 are presented the values of the ground wire impedance Z_{cp_d} and the

Table 1. Ground wire impedance and mutual impedance, per one span for different values of the soil resistivity.

$\rho\,[\Omega m]$	10	50	100	200	300	400	500
$R_{cp_d}\,[\Omega]$	0.190	0.201	0.206	0.210	0.214	0.216	0.217
$Z_m\,[\Omega]$	0.0578	0.0695	0.0747	0.0800	0.0831	0.0853	0.0870

mutual impedance Z_m, per one span, between the ground wire and the faulted phase, calculated with Carson's formulas, for different values of the soil resistivity.

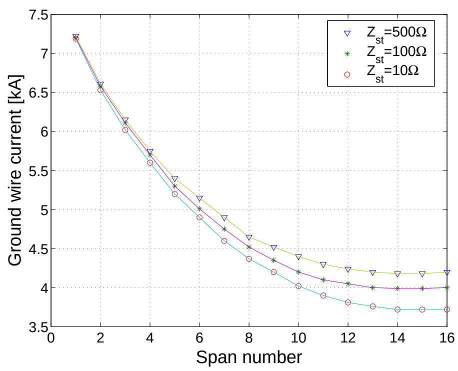

Fig. 8. Current flowing in the ground wire in case of different values of soil resistivity; $N = M = 15$ towers between faulted tower and the both ends of the line; $Z_{st} = 10\Omega$.

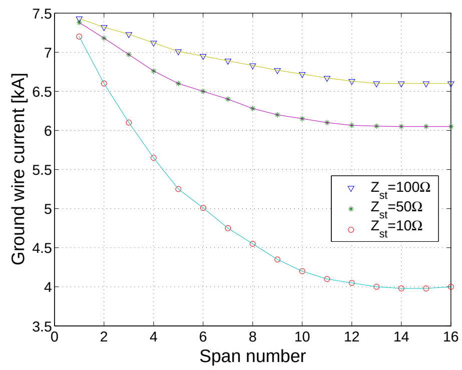

Fig. 9. Current flowing in the ground wire in case of different values for towers impedances; $N = M = 15$ towers between faulted tower and the both ends of the line.

Using the same values of the ground wire and mutual impedance calculated for different values of soil resistivity given in the Table 1, the currents flowing in the ground wire in case of the fault at any tower of the line are shown in Figure 8.

Figure 9 shows the currents flowing in the ground wire for different values of the towers impedances. The total fault current from both stations was assumed to be $I_d = 15000A$ and $I_d^{'} = I_d^{''} = 7500A$. Those values are valid for a soil resistivity of $100\Omega m$. It was assumed that the fault appears at the middle tower of the line, so there are $N = 15$ towers and respectively $M = 15$ towers between the faulted tower and the terminals.

4 Conclusions

This paper describes an analytical method in order to determine the ground fault current distribution in power networks, when the fault appears at the last tower of the line, respectively at any tower of the transmission line. The author developed this model based on the earlier approach of Rudenberg [7] and Verma [8].

Rudenberg developed a model that was valid only for long lines, without taking into account the mutual coupling between the faulted phase and the ground conductor. Verma developed that model by taking into account the mutual coupling, but he treats only the case of long line to. The authors improved their model by considering the case of a short line, when it has to consider the station from the end of the line.

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