# About the Coupling Factor Influence on the Ground Fault Current Distribution on Overhead Transmission Lines

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*Abstract*— A phase-to-ground fault occurring on a transmission line divides the line into two sections, each extending from the fault towards one end of the line. These two sections of the line may be considered infinite if some certain conditions are met; otherwise, they must be regarded as finite. This paper treats the case when those two sections of the line are both very long and allows the determination of the ground fault current distribution in power networks. The influence of the coupling factor between the faulted phase and the ground wire on the ground fault current distribution is studied.

Index Terms—fault currents, power network, transmission lines

#### I. INTRODUCTION

HIGH-VOLTGAE systems have an effectively grounded neutral. When a ground fault occurs on an overhead transmission line in a three-phase power network with grounded neutral, the fault current returns to the grounded neutral through the towers, ground return path and ground wires.

Knowledge of fault current distribution is important for the size selection of an overhead ground wire, and respectively for the evaluation of the faulted tower's voltage rise. In electromagnetic interference problems, the best way to investigate the soil's behavior as conducting media is to determine the current distribution within ground [6].

The ground fault divides the line into two sections, each extending from the fault towards one end of the line. Depending of the number of towers between the faulted tower and the stations, respectively of the distance between the towers, these two sections of the line may be considered

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infinite, in which case the ground fault current distribution is independent on the termination of the network; otherwise, they must be regarded as finite, in which case the ground fault current distribution may depend greatly on the termination of the network.

Based on Kirchoff's theorems and on some methods presented by Rudenberg [8], Verma [9], Endreny [4], Edelmann [3] and Dawalibi [2], an analytical method in order to determine the ground fault current distribution in effectively grounded power network, was already presented in previous works [10]-[11].

In [11] it was presented the case when the fault appears to the last tower of the transmission line, considering both infinite and finite transmission line, respectively the case when the fault appears at any tower of the transmission line, the two sections of the line are finite and it is assumed that the fault is fed from both directions. The method is applied to three phase systems with mutual coupling between phase conductors and ground wire.

In this paper it will be developed and analyzed the case when the fault appears at large distance from both terminals, and the two sections of the line between the fault and the terminals could be considered infinite.

The calculation method is based on the following assumptions: impedances are considered as lumped parameters in each span of the transmission line, line capacitances are neglected; the contact resistance between the tower and the ground wire, and the tower resistance between the ground wire and the faulted phase are neglected; the network is considered linear in the sinusoidal steady-state.

In the following section it will be shown how might be obtained the expressions of the currents flowing to ground through the towers and the currents in every span of the ground conductor. Furthermore, the influence of the coupling factor between the faulted phase and the ground wire on the ground fault current distribution is studied, too.

It is considered an overhead transmission line with one ground wire, connected to the ground at every tower of the line and the fault appears at one tower located at very large distance from both terminals.

#### II. FAULTS ON OVERHEAD LINE

Figure 1 presents the connection of a ground wire to earth through transmission towers. It is assumed that all the transmission towers have the same ground impedance  $Z_{st}$  and the distance between towers is long enough to avoid the influence between their grounding electrodes.

The self-impedance of the ground wire connected between two grounded towers, called the self-impedance per span, was noted with  $Z_{cp_A}$ .

It was assumed that the distance between two consecutive towers is the same for every span.  $Z_{cp_m}$  represents the mutual-impedance between the ground wire and the faulted phase conductor, per span.

As a first step and without loosing the generality, it is assumed that the fault occurs at the last tower. The half-line will be studied first, and then the full-line analysis will be accomplished by regarding it as a composite of two half-lines. When the fault appears, part of the ground fault current will get to the ground through the faulted tower, and the rest of the fault current will get diverted to the ground wire and other towers.



Figure 1. Fault current distribution

As it was already presented in [11], the current  $I_n$  flowing to ground through the *n*-th tower, counted from the terminal tower where the fault is assumed to take place, is equal with the difference between the currents  $i_n$  and  $i_{n+1}$ :

$$I_n = i_n - i_{n+1} \tag{1}$$

The loop equation for the *n*-th mesh is given by the next expression:

$$I_{n}Z_{st} - I_{n-1}Z_{st} + i_{n}Z_{cp_{d}} - vI_{d}Z_{cp_{d}} = 0$$
<sup>(2)</sup>

v in expression (2) represents the coupling factor between

the overhead phase and ground wire  $(V = \frac{Z_{cp_m}}{Z_{cp_d}})$ .

The equation (2) could be written in the next form:

$$i_n = \frac{(I_{n-1} - I_n)Z_{st}}{Z_{cp_d}} + vI_d$$
(3)

Similarly:

$$i_{n+1} = \frac{(I_n - I_{n+1})Z_{st}}{Z_{cp_d}} + vI_d$$
(4)

Substituting equations (3) and (4) in equation (1), for the current in the faulted tower will be obtained the next equation, which is a second order difference equation:

$$I_n \frac{Z_{cp_d}}{Z_{st}} = I_{n+1} - 2I_n + I_{n-1}$$
(5)

According to [8], the sollution of this equation is:

$$I_n = Ae^{\alpha n} + Be^{-\alpha n} \tag{6}$$

According with the solution (6) which contains the arbitrary parameters A and B, the current flowing to ground through successive towers, has an exponentially variation. The arbitrary parameters A and B could be obtained from the boundary conditions.

Parameter  $\alpha$  in the solution (6) could be obtained by substituting the solution (6) in equation (5). Because  $Z_{cp_d} \ll Z_{st}$ , it can be written:

$$\alpha \approx \sqrt{\frac{Z_{cp_d}}{Z_{st}}} \tag{7}$$

By applying equation (1) to the (n-1) tower, it will be obtained the following expression:

$$I_{n-1} = i_{n-1} - i_n \tag{8}$$

By substituting the equations (1) and (8) in equation (2), it will be obtained the next equation with a constant term:

$$i_n \frac{Z_{cp_d}}{Z_{st}} = i_{n+1} - 2i_n + i_{n-1} + vI_d \frac{Z_{cp_d}}{Z_{st}}$$
(9)

Similar with equation (5), the current in the ground wire is given by the next solution:

$$i_n = ae^{\alpha n} + be^{-\alpha n} + V I_d \tag{10}$$

In expression (10), a, b represents the arbitrary parameters. Because of the link between currents  $i_n$  and  $I_n$ , the arbitrary parameters A, B and a, b are not independent. By substituting the solutions (6) and (10) in equation (1), it will be obtained:

$$Ae^{\alpha n} + Be^{-\alpha n} = ae^{\alpha n}(1-e^{\alpha}) + be^{-\alpha n}(1-e^{-\alpha})$$
(11)

Because these relations are the same for every value of n, it will be obtained the next expressions:

$$A = a(1 - e^{\alpha}) \tag{12}$$

$$B = b(1 - e^{-\alpha}) \tag{13}$$

The current in the ground wire will be then given by the following expression:

$$i_{n} = A \frac{e^{\alpha n}}{1 - e^{\alpha}} + B \frac{e^{-\alpha n}}{1 - e^{-\alpha}} + \nu I_{d}$$
(14)

The boundary condition at the terminal tower of figure 1 is:

$$I_d = I_0 + i_1$$
(15)

That means that the fault current is given by the sum between the current in the faulted tower and the current in the first span of the ground wire.

In case that it is considered that the line is sufficiently long so that, after some distance, the varying portion of the current exponentially decays to zero, then the parameter  $A \rightarrow 0$ . In this case only the parameter *B* must be found from the boundary conditions [9]. According to (6) and (14), results:

$$I_n = Be^{-\mathcal{O}n} \tag{16}$$

$$i_n = B(e^{-\alpha n} / 1 - e^{-\alpha}) + vI_d$$
<sup>(17)</sup>

Substituting these expressions in (15), with n = 0 for  $I_n$  and n = 1 for  $i_n$ , it will be obtained:

$$I_{d} = B + B(e^{-\alpha} / 1 - e^{-\alpha}) + vI_{d}$$
(18)

For *B* it will be obtained the next expression:

$$B = (1 - \nu)(1 - e^{-\alpha})I_d$$
(19)

The current in the faulted tower will get the expression:

$$I_0 = B = (1 - v)(1 - e^{-\alpha})I_d$$
(20)

The current in the first span, counted from the faulted tower, will be:

$$i_1 = I_d - I_0 = I_d [e^{-\alpha} + \nu(1 - e^{-\alpha})]$$
(21)

The voltage rise at the terminal tower is:

$$U_0 = Z_{st} I_0 = Z_{st} (1 - \nu)(1 - e^{-\alpha}) I_d$$
(22)

A long line is one which has a middle section where the line is divided by each tower into infinite half-lines. Next, it is treated the case when those two sections of the line are both long.

In figure 2 the fault occurs at large distance from both terminals, at the tower number 0. The fault divides the line into two sections, each extending from the fault towards one end of the line.



Figure 2. Fault current distribution

The boundary condition at the faulted tower will be:

$$I_{d} = I_{0} + 2i_{1} \tag{23}$$

Substituting expressions (16) and (17) in (23), with n = 0 for  $I_n$  and n = 1 for  $i_n$ , it will be obtained:

$$I_d = B + 2(B\frac{e^{-\alpha}}{1 - e^{-\alpha}} + vI_d)$$
<sup>(24)</sup>

Then, the current in the faulted tower will get the expression:

$$I_0 = B = \frac{1 - e^{-\alpha}}{1 + e^{-\alpha}} (1 - 2\nu) I_d$$
(25)

The current in the first span, counted from the faulted tower, will be the following:

$$i_1 = \frac{I_d - I_0}{2} \tag{26}$$

The voltage at the faulted tower will be:

$$U_{0} = Z_{st}I_{0} = Z_{st}(1 - 2\nu)I_{d}th\frac{\alpha}{2}$$
(27)

If the the coupling factor between the overhead phase and ground wire is neglected, then the current in the faulted tower will get the following expression:

$$I_0 = B = I_d th \frac{\alpha}{2}$$
(28)

The voltage at the faulted tower will be:

$$U_{0} = Z_{st}I_{0} = Z_{st}I_{d}th\frac{\alpha}{2} = ZI_{d}$$
<sup>(29)</sup>

With Z in expression (29) was noted the equivalent impedance of the network looking back from the fault.

#### III. RESULTS

In order to illustrate the theoretical approach outlined in section above, it is considering that the line who connects two stations is a 110kV transmission line with aluminium-steel 185/32mm<sup>2</sup> and one aluminium-steel ground wire 95/55mm<sup>2</sup> (figure 3) [5].



The line impedances per one span are determined on the bases of the following assumptions: the average span's length is 250m; the resistances per unit length of ground wire is  $0.3 \Omega / km$  and it's diameter is 16mm.

Ground wire impedance per one span  $Z_{cp_d}$  and the mutual impedance  $Z_m$  between the ground wire and the faulted phase are calculated for different values of the soil resistivity  $\rho$  with formulas based on Carson's theory of the ground return path [1].

Impedance  $Z_m$  is calculated only in relation to the faulted

phase conductor, because it could not be assumed that a line section of a few spans is transposed.

The fault was assumed to occur on the phase which is the furthest from the ground conductors, because the lowest coupling between the phase and ground wire will produce the highest tower voltage.

The total fault current from both stations was assumed to be  $I_d = 15000$  A. Those values are valid for a soil resistivity of 100  $\Omega$  m.

It was assumed that the fault appears at the middle tower of the line, so there are N=20 towers between the faulted tower and each of the terminals.

All the further presented quantitative results are based on the theoretical approaches developed during the previous section. In order to do this there were developed some numerical intensive programs written in Matlab 7.0 software frame.

Figure 4 shows the currents flowing in the transmission line towers ( $Z_{st} = 4 \Omega$ ), first considering the mutual coupling between the faulted phase and the ground conductor, than neglecting that mutual coupling.



It can be seen that in the absence of mutual coupling, the

fault current will flow through the ground through a smaller number of towers than in the mutual coupling presence. Figure 5 shows the currents flowing in the ground wire for

 $Z_{st}$  = 4  $\Omega$ , first considering the mutual coupling between the faulted phase and the ground conductor, than neglecting that mutual coupling.

It can be seen that in the absence of coupling between the overhead phase and ground wire, the total fault current will gradually flow into the ground through the towers, and, if the line is long enough, no current remains in the overhead wire. In contrast, in the presence of coupling factor a portion of the current will stay in the overhead wire.



The voltage rise of the faulted tower depends of a number of factors. Some of the most important factors are: magnitudes of fault currents on both sides of the fault location, fault location with respect to the line terminals, conductor arrangement on the tower and the location of the faulted phase, the ground resistance of the faulted tower, soil resistivity, number, material and size of ground wires.

Figure 6 shows the influence of the mutual impedance between the faulted phase and the ground wire on the voltage rise of faulted tower. It was considered that the ground wire impedance is  $Z_{cp_d} = 0.19\Omega$ .



Figure 6. Voltage rise of the faulted tower as a function of the tower ground impedance

In order to see the effct of the mutual coupling between the faulted phase and the ground wire, in figure 7 is represented the impedance of the infinite line from equation (29), for a ground wire impedance  $Z_{cp_d} = 0.19\Omega$ , as a function of the tower ground impedance, considering, respective neglecting the coupling factor.



Figure 7. Equivalent impedance of the infinite

In order to validate the proposed analytical expressions their results were compared with those measured in similar conditions using real fault tests [2]. Also the developed analytical methods were compared with other recognized analytical methods [7]. In both cases the obtained results were very similarly with those obtained by other researchers. For example, the faulted tower's voltage rise error is always smaller than 5%. As important advantages, the presented method is simpler and far less time consuming than others.

## IV. CONCLUSION

A parametric analysis is done in order to study the effects of the coupling factor between the faulted phase and the ground wire on the ground fault current distribution in power networks. It was considered an overhead transmission line with one ground wire, connected to the ground at every tower of the line. It was assumed that the fault occurs at large distance from both terminals. There were presented the expressions of the currents flowing to ground through the towers and the currents in every span of the ground conductor. These currents are varying exponentially and their expressions contain arbitrary parameters which could be obtained from the boundary conditions. For the long lines case, one of these parameters could be zero. It is apparent that in the absence of coupling between the overhead phase and ground wire the total fault current will gradually flow into the ground through the towers, and, if the line is long enough, no current remains in the overhead wire; whereas in the presence of coupling factor a portion of the current will stay in the overhead wire.

The mutual impedance between the ground conductor and the faulted phase conductor reduces the total circuit impedance. Neglecting this mutual impedance the fault current would be significantly higher.

From the model presented above, it can be seen that due to the mutual coupling, the fault current is reduced with  $(1 - \nu)$  factor. It also can be seen that in the absence of mutual coupling, the fault current will flow through the ground

through a smaller number of towers than in the mutual coupling presence. As a consequence, in this case the faulted tower's voltage rise and also its neighbours' voltage rises will be higher in an artificial manner. In conclusion our developed model involves more accurate realistic analysis of the power transmission networks.

The paper presents a practical method for ground fault analysis being extremely useful in designing grounding systems. The method can be also used for the design of new transmission lines in order to select the size of the ground wire and for evaluating safety conditions near transmission towers.

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