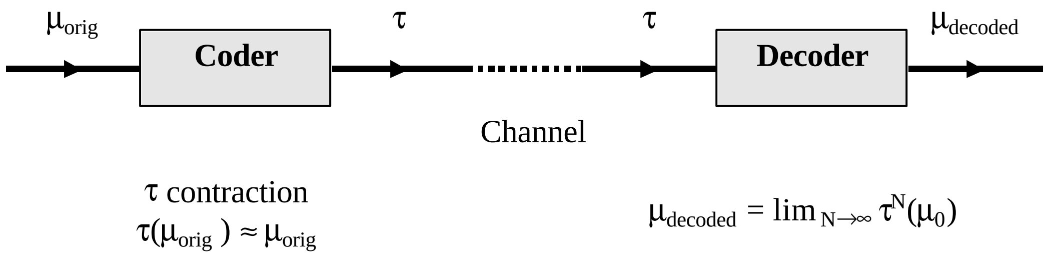

Figura 4 General coding - decoding scheme

Computing of transformation parameters: scale (s) and offset (o)

In the following we will try to get the optimum values for the parameters of the luminance part of the transform, considering the geometric transform already applied (domain choosed and scaled).

Considering two image block D and R as in Figure 11, having the intensities of pixels $d_1,d_2,...,d_n$ and $r_1,r_2,...,r_n$, respectively, we have to minimize the following (the Euclidian distance):

$$E(D,R,s,o)^2 = \sum_{i=1}^{n}\left[r_i - \left(s\cdot d_i + o\right)\right]^2 \qquad (11)$$

Because $E(D,R,s,o)^2$ is a order 2 function related to s and o, having the coefficient of order 2 term positive, consequently having a minimum, in order to get the minimum we have to make the derivatives regarding s and o zero:

$$\begin{cases} \dfrac{\partial E(D,R,s,o)^2}{\partial s} = 0 \\ \dfrac{\partial E(D,R,s,o)^2}{\partial o} = 0 \end{cases} \qquad (12)$$

Therefore, using for $E(D,R,s,o)^2$ the expression (11) we get

$$\begin{cases} \sum_{i=1}^{n} 2\left[r_i - (s\cdot d_i + o)\right](-d_i) = 0 \\ \sum_{i=1}^{n} 2\left[r_i - (s\cdot d_i + o)\right] = 0 \end{cases} \qquad (13)$$

Separating s and o we get the system:

$$\begin{cases} s\cdot\sum_{i=1}^{n} d_i^{\,2} + o\cdot\sum_{i=1}^{n} d_i = \cdot\sum_{i=1}^{n} r_i d_i \\ s\cdot\sum_{i=1}^{n} d_i + o\cdot n = \cdot\sum_{i=1}^{n} r_i \end{cases} \qquad (14)$$

We can now get easy the value for s:

$$s = \frac{n\cdot\left(\sum_{i=1}^{n} r_i d_i\right) - \left(\sum_{i=1}^{n} r_i\right)\cdot\left(\sum_{i=1}^{n} d_i\right)}{n\cdot\left(\sum_{i=1}^{n} d_i^{\,2}\right) - \left(\sum_{i=1}^{n} d_i\right)^2} \qquad (15)$$

and for o:

$$o = \frac{1}{n}\left[\left(\sum_{i=1}^{n} r_i\right) - s\cdot\left(\sum_{i=1}^{n} d_i\right)\right] \qquad (16)$$

Certainly, if the denominator in expression (15) is zero, we consider

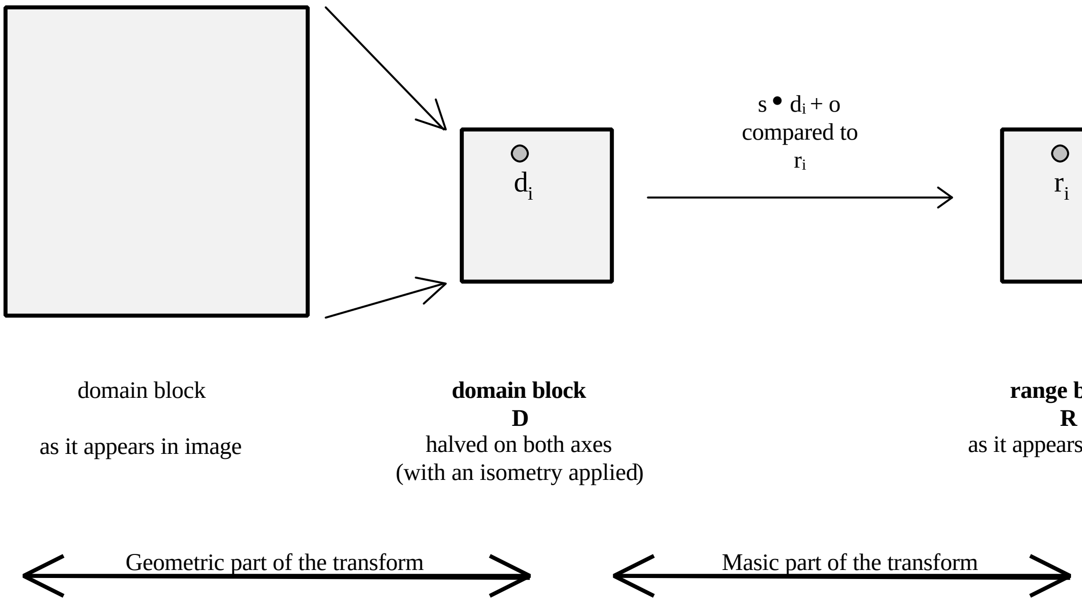

Figure 11 Context for computing the s and o parameters of the transform

$$s = 0 \text{ si } o = \frac{1}{n}\left(\sum_{i=1}^{n} r_i\right) \tag{17}$$

With s and o already computed, the expression for the error $E(D,R,s,o)^2$ becomes:

$$E(D,R)^2 = \frac{1}{n}\left\{\left(\sum_{i=1}^{n} r_i^2\right) + s\cdot\left[s\cdot\left(\sum_{i=1}^{n} d_i^2\right) - 2\cdot\left(\sum_{i=1}^{n} r_i d_i\right) + o\cdot 2\cdot\left(\sum_{i=1}^{n} d_i\right)\right] + o\cdot\left[o\cdot n - 2\cdot\left(\sum_{i=1}^{n} r_i\right)\right]\right\} \tag{18}$$

Certainly, to achieve compression, we will use a reduced number of bits for the values of s and o. Therefore the values are uniformly quantized getting $\bar{s}$ and $\bar{o}$ (usually with 5 and 7 bits respectively).

Decoding

Decoding consists in applying iteratively the fractal transformation (determined at coding and now taken from the compressed stream) on an input image and obtaining another transformed image. At the next iteration the transformed image obtained in the previous step becomes input image for the next step. After a number of steps, the iterative process converges. The input image for the first step can be choosed.

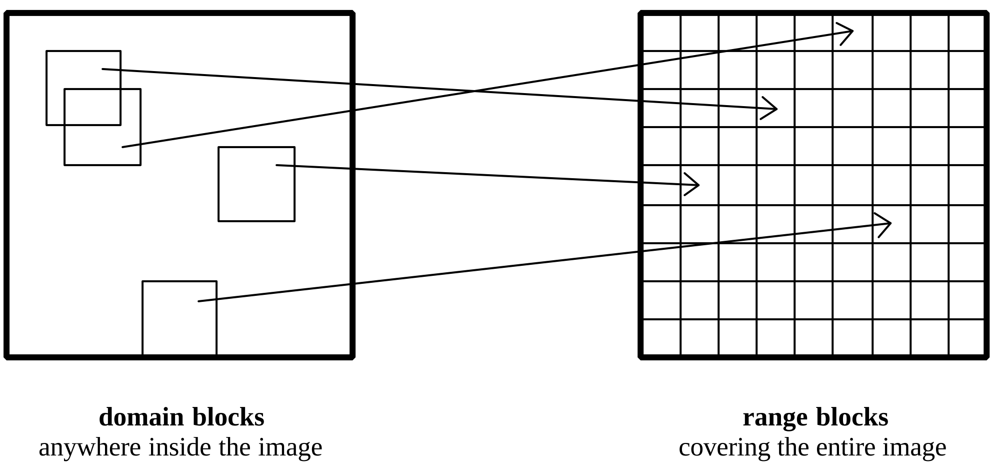

Figure 7 Domain → Range mapping (also applied iteratively at decoding)

The 8 isometries

Usually, as part of he geometric transform, the 8 isometric transformations of a block (of dimension B*B) are taken into consideration. These are:

1. identity

   $(i_1\mu)_{y,x} = \mu_{y,x}$
2. reflection about vertical axis

   $(i_2\mu)_{y,x} = \mu_{y,B-1-x}$
3. Reflection about horizontal axis

   $(i_3\mu)_{y,x} = \mu_{B-1-y,x}$
4. Reflection about first diagonal

   $(i_4\mu)_{y,x} = \mu_{x,y}$
5. Reflection about second diagonal

   $(i_5\mu)_{y,x} = \mu_{B-1-x,B-1-y}$
6. Rotation around center with 90°

   $(i_6\mu)_{y,x} = \mu_{y,B-1-x}$
7. Rotation around center with 180°

   $(i_7\mu)_{y,x} = \mu_{B-1-y,B-1-x}$
8. Rotation around center with 270°

   $(i_8\mu)_{y,x} = \mu_{B-1-y,x}$

A graphic representation of these isometries is given in Figure 8 and in figure 9 examples of their applying on the whole Lena image are given (in fact the isometries are applied on a block level not on the entire image level). Enlarging the dimension of the "domain pool" by considering the 8 isometries for quality improvement is an option unanimous accepted.

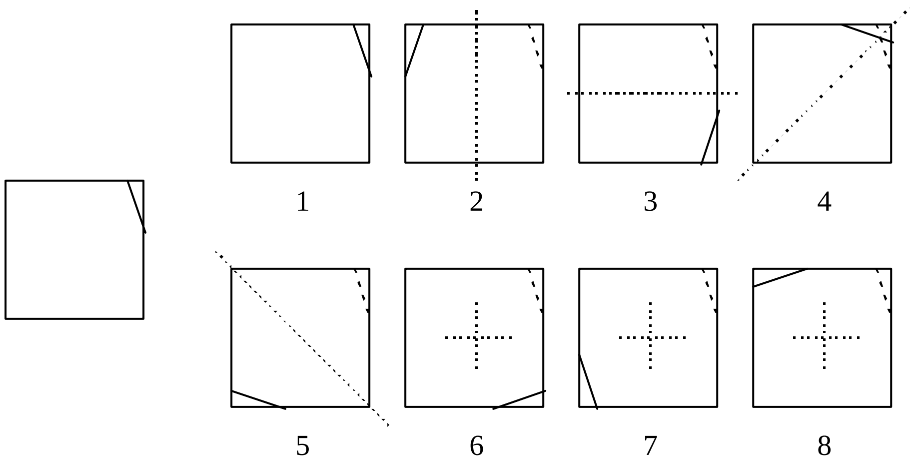

Figure 8 The 8 isometries

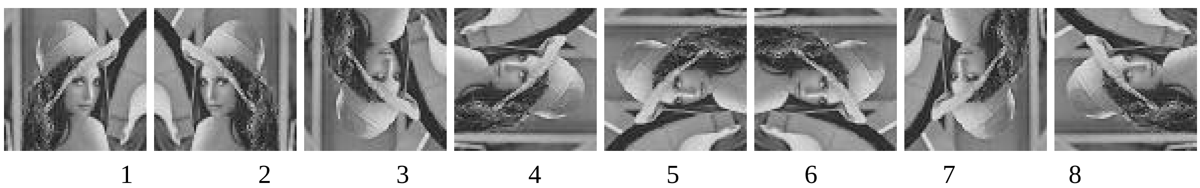

Figure 9 Example of applying the isometries